

$$a) f(x) = \frac{x^2+2x}{x^2-4}$$

$$\text{Dom}(f) = \mathbb{R} - \{-2, 2\}$$

Continuidad: $f(x)$ es cont. en su dominio.

* $x = -2$:

$$\lim_{x \rightarrow -2^-} \frac{x^2+2x}{x^2-4} = \lim_{x \rightarrow -2^-} \frac{x(x+2)}{(x+2)(x-2)} = \frac{-2}{-4} = \frac{1}{2}$$

$$\lim_{x \rightarrow -2^+} \frac{x^2+2x}{x^2-4} = \frac{1}{2}$$

$\Rightarrow$ En $x = -2$ $\nexists$ discontinuidad EVITABLE.

* $x = 2$:

$$\lim_{x \rightarrow 2^-} \frac{x^2+2x}{x^2-4} = \frac{8}{0^-} = -\infty$$

$$\lim_{x \rightarrow 2^+} \frac{x^2+2x}{x^2-4} = \frac{8}{0^+} = +\infty$$

$\left\{ \begin{array}{l} \text{En } x=2 \text{ } \nexists \text{ discontinuidad} \\ \text{de SALTO INFINITO.} \end{array} \right.$

Asintotas:

Asintota vertical en $x = 2$.

Asintotas horizontales:

$$\lim_{x \rightarrow +\infty} \frac{x^2+2x}{x^2-4} = 1 \quad \left\{ \begin{array}{l} y=1 \text{ es asintota horizontal.} \end{array} \right.$$

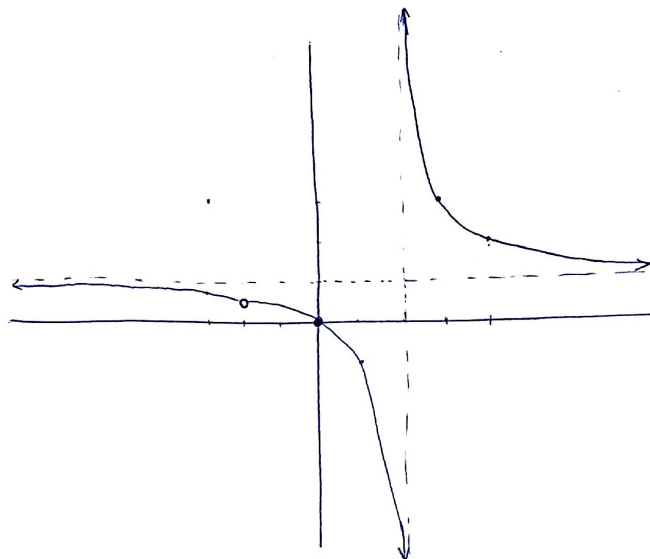
$$\lim_{x \rightarrow -\infty} \frac{x^2+2x}{x^2-4} = 1$$

Puntos de corte con los ejes:

Eje Y: $x=0$ $\left\{ \begin{array}{l} \text{corta en } (0,0) \\ f(0)=0 \end{array} \right.$

Eje X: $y=0$

$$\frac{x^2+2x}{x^2-4} = 0 \Rightarrow x(x+2) = 0 \Rightarrow \left\{ \begin{array}{l} x=0 \rightarrow \text{corta en } (0,0) \\ x=-2 \rightarrow \nexists f(-2) \end{array} \right.$$



x	y
1	-1
3	3
-3	-3/5
4	2

$$b) f(x) = \begin{cases} \frac{5}{x-2} & ; x \leq 3 \\ 4x+5 & ; x > 3 \end{cases}$$

$$\text{Dom}(f) = \mathbb{R} - \{2\}$$

Continuidad:

* $x=2$:

$$\lim_{x \rightarrow 2^-} \frac{5}{x-2} = \frac{5}{0^-} = -\infty \quad \left\{ \begin{array}{l} \text{En } x=2. \text{ Discontinuidad} \\ \text{de SALTO INFINITO.} \end{array} \right.$$

$$\lim_{x \rightarrow 2^+} \frac{5}{x-2} = \frac{5}{0^+} = +\infty$$

* $x=3$:

$$\begin{aligned} f(3) &= 5 \\ \lim_{x \rightarrow 3^-} f(x) &= 5 \\ \lim_{x \rightarrow 3^+} f(x) &= 7 \end{aligned} \quad \left\{ \begin{array}{l} \text{En } x=3. \text{ Discontinuidad de} \\ \text{SALTO FINITO.} \end{array} \right.$$

Asintotas:

Asintota vertical en $x=2$.

Asintotas horizontales:

$$\lim_{x \rightarrow +\infty} f(x) = +\infty \quad \left| \quad \underline{y=0} \text{ es asintota horizontal.} \right.$$

$$\lim_{x \rightarrow -\infty} f(x) = 0$$

Puntos de corte con los ejes:

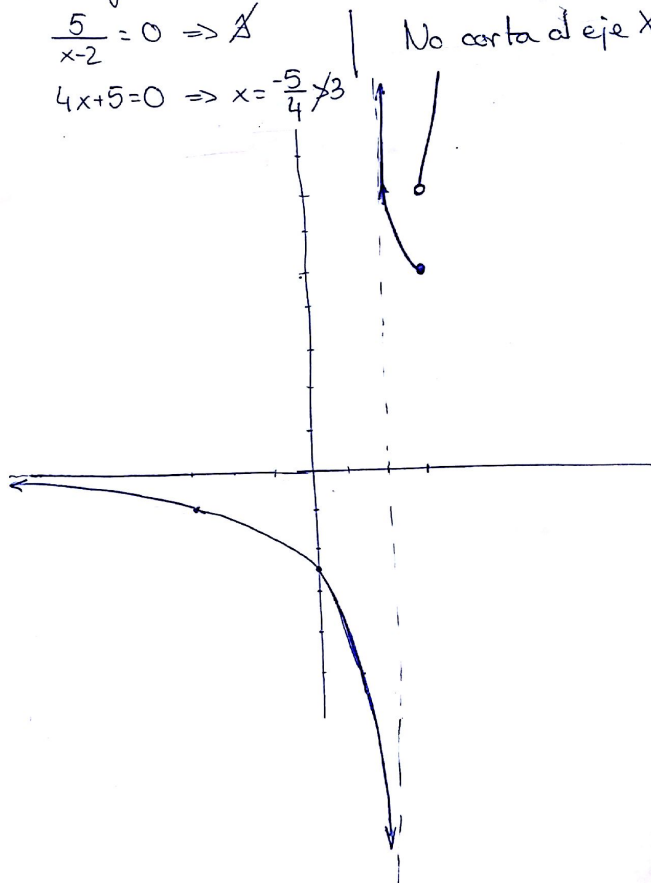
$$\begin{aligned} \text{Eje Y: } x=0 \\ f(0) = \frac{-5}{2} \end{aligned} \quad \left\{ \begin{array}{l} \text{corta en } (0, \frac{-5}{2}) \end{array} \right.$$

$$\text{Eje X: } y=0$$

$$\frac{5}{x-2} = 0 \Rightarrow \text{No}$$

$$4x+5=0 \Rightarrow x = -\frac{5}{4} \neq 3$$

No corta al eje X.



$$c) f(x) = \begin{cases} 2^{\frac{1}{x+1}} & ; x \leq 0 \\ 3x^2 - 2 & ; x > 0 \end{cases}$$

$$\text{Dom}(f) = \mathbb{R} - \{-1\}$$

Continuidad:

* $x = -1$:

$$\begin{aligned} \lim_{x \rightarrow -1^-} 2^{\frac{1}{x+1}} &= 2^{\frac{1}{0^-}} = 2^{-\infty} = 0 \\ \lim_{x \rightarrow -1^+} 2^{\frac{1}{x+1}} &= 2^{\frac{1}{0^+}} = 2^{+\infty} = +\infty \end{aligned} \quad \left\{ \begin{array}{l} \text{En } x = -1. \text{ Discontinuidad} \\ \text{de SALTO INFINITO.} \end{array} \right.$$

* $x = 0$:

$$\begin{aligned} f(0) &= \frac{1}{2} \\ \lim_{x \rightarrow 0^-} f(x) &= \frac{1}{2} \\ \lim_{x \rightarrow 0^+} f(x) &= -2 \end{aligned} \quad \left\{ \begin{array}{l} \text{En } x = 0. \text{ Discontinuidad de} \\ \text{SALTO FINITO.} \end{array} \right.$$

Asintotas:

Asintota vertical en $x = -1$.

Asintotas horizontales:

$$\lim_{x \rightarrow +\infty} f(x) = +\infty$$

$$\lim_{x \rightarrow -\infty} f(x) = 1 \Rightarrow y = 1 \text{ es asintota horizontal.}$$

Puntos de corte con los ejes:

$$\begin{aligned} \text{Eje Y: } x &= 0 \quad \left\{ \begin{array}{l} f(0) = \frac{1}{2} \\ (0, \frac{1}{2}) \end{array} \right. \end{aligned}$$

$$\text{Eje X: } y = 0$$

$$\begin{aligned} 2^{\frac{1}{x+1}} &= 0 \rightarrow \text{No tiene solución} \\ 3x^2 - 2 &= 0 \rightarrow x = \pm \sqrt{\frac{2}{3}} \Rightarrow \left(\sqrt{\frac{2}{3}}, 0 \right) \end{aligned}$$

