1. $f(x) = xe^{1/x}$; $x \neq 0$.

Asíntotas verticales:

$x=0$: $\lim_{x\to 0^+} xe^{1/x} = 0\cdot\infty \Rightarrow \lim_{x\to 0^+} \frac{e^{1/x}}{1/x} = \frac{\infty}{\infty}$ → Resolvemos la indeterminación aplicándola Regla de L'Hôpital.

$$\lim_{x\to 0^+} \frac{e^{1/x}\cdot\left(\frac{-1}{x^2}\right)}{-1/x^2} = \lim_{x\to 0^+} e^{1/x} = \underline{\underline{+\infty}} \Rightarrow \boxed{\text{Hay asíntota vertical en } x=0}$$

$$\lim_{x\to 0^-} xe^{1/x} = 0\cdot e^{-\infty} = 0.$$

Asíntotas horizontales:

$$\left.\begin{aligned}&\lim_{x\to+\infty} xe^{1/x} = +\infty\\ &\lim_{x\to-\infty} xe^{1/x} = -\infty\end{aligned}\right\}$$ No hay asíntotas horizontales.

Asíntotas oblicuas: $y = mx+n$.

$$m = \lim_{x\to+\infty} \frac{xe^{1/x}}{x} = \lim_{x\to+\infty} e^{1/x} = 1.$$

$$n = \lim_{x\to+\infty} (xe^{1/x} - x) = \lim_{x\to+\infty} (e^{1/x}-1)x = 0\cdot\infty \Rightarrow \lim_{x\to+\infty} \frac{e^{1/x}-1}{1/x} = \frac{0}{0}$$ → Aplicamos L'Hôpital

$$\lim_{x\to+\infty} \frac{e^{1/x}\cdot(-1/x^2)}{-1/x^2} = \lim_{x\to+\infty} e^{1/x} = 1 \qquad \Rightarrow y = x+1$$

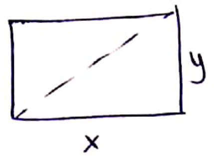
$$m' = \lim_{x\to-\infty} \frac{xe^{1/x}}{x} = \lim_{x\to-\infty} e^{1/x} = 1$$

$$n' = \lim_{x\to-\infty} (xe^{1/x} - x) = \lim_{x\to-\infty} \frac{e^{1/x}-1}{1/x} = \frac{0}{0} \to \text{L'Hôpital}: \lim_{x\to-\infty} \frac{e^{1/x}\cdot(-1/x^2)}{-1/x^2} = 1$$

$$\Rightarrow y = x+1$$

$$\boxed{\text{Hay asíntota oblicua en } y = x+1.}$$

2.



Perímetro: $8 = 2x + 2y \Rightarrow y = 4 - x$

Diagonal: $d(x,y) = \sqrt{x^2 + y^2} \Rightarrow d(x) = \sqrt{x^2 - (4-x)^2}$

$$d(x) = \sqrt{2x^2 - 8x + 16}$$

$$d'(x) = \frac{4x-8}{2\sqrt{2x^2-8x+16}} = \frac{2x-4}{\sqrt{2x^2-8x+16}} = 0 \Rightarrow 2x-4=0 \Rightarrow \boxed{x=2}$$

$$\Rightarrow \boxed{y = 4-2 = 2.}$$

El rectángulo de perímetro 8 cm de menor diagonal es el cuadrado de lado 2cm.

3. $f(x) = ax^3 + bx^2 + cx + d$

$f'(1) = 0 \longrightarrow f'(x) = 3ax^2 + 2bx + c \rightarrow f'(1) = 3a + 2b + c = 0$ ①

$f(0) = 1 \longrightarrow d = 1$

$f''(0) = 0 \longrightarrow f''(x) = 6ax + 2b \rightarrow f''(0) = 2b = 0 \Rightarrow b = 0$

$\int_0^1 f(x)\,dx = \frac{9}{4} \rightarrow \int_0^1 (ax^3 + cx + 1)\,dx = \left(\frac{a}{4}x^4 + \frac{c}{2}x^2 + x\right\Big|_0^1 = \frac{a}{4} + \frac{c}{2} + 1 = \frac{9}{4}$ ②

① $\rightarrow 3a + c = 0$

② $\rightarrow a + 2c = 5$

$$\left\{\begin{array}{l} 3a + c = 0 \\ -3a - 6c = -15 \end{array}\right.$$

$-5c = -15 \Rightarrow c = 3 \rightarrow a = -1$

$\boxed{a = -1;\ b = 0;\ c = 3;\ d = 1}$

$f(x) = -x^3 + 3x^2 + 1$

Opción A.

4. $f(x)=\begin{cases} x^2+ax+b \,; & 0\leq x<2 \\ cx+1 \,; & 2\leq x\leq 4 \end{cases}$

a) $f(x)$ es continua $\Rightarrow \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

$4+2a+b = 2c+1 \Rightarrow \underline{2a+b-2c=-3}$ ①

$f(x)$ es derivable $\Rightarrow \lim_{x\to 2^-} f'(x) = \lim_{x\to 2^+} f'(x)$

$f'(x)=\begin{cases} 2x+a \,; & 0<x<2 \\ c \,; & 2<x<4 \end{cases}$ $\qquad \underline{4+a=c}$ ②

$f(0)=f(4) \Rightarrow \underline{b=4c+1}$ ③

$\begin{cases} 2a+b-2c=-3 \\ 4+a=c \rightarrow a=c-4 \\ b=4c+1 \end{cases} \rightarrow 2(c-4)+4c+1-2c=-3$

$2c-8+4c+1-2c=-3$

$4c=4 \Rightarrow \boxed{\begin{array}{l} c=1 \\ a=-3 \\ b=5 \end{array}}$ $f(x)=\begin{cases} x^2-3x+5 & 0\leq x<2 \\ x+1 & 2\leq x\leq 4 \end{cases}$

b) $f'(x)=\begin{cases} 2x-3 \,; & 0<x<2 \\ 1 \,; & 2<x<4 \end{cases}$

$f'(x)=0 \Rightarrow 2x-3=0 \Rightarrow x=\frac{3}{2}$

$\boxed{\text{La derivada se anula en } x=\frac{3}{2}.}$

5. $\int_0^1 x \ln(x+1)\, dx$

Integramos por partes: $u = \ln(x+1) \to du = \frac{1}{x+1} dx$

$dv = x\,dx \to v = \frac{x^2}{2}$

$$\int_0^1 x\ln(x+1)\,dx = \left[\frac{x^2}{2}\ln(x+1)\right]_0^1 - \int_0^1 \frac{x^2}{2(x+1)}\,dx =$$

$$\frac{x^2}{x+1} = x-1+\frac{1}{x+1}$$

$$= \left[\frac{x^2}{2}\ln(x+1)\right]_0^1 - \frac{1}{2}\left[\int_0^1 (x-1)\,dx + \int_0^1 \frac{1}{x+1}\,dx\right] = \left[\frac{x^2}{2}\ln[x+1] - \frac{1}{2}\cdot\left[\frac{x^2}{2} - x + \ln(x+1)\right]\right]_0^1 =$$

$$= \frac{1}{2}\ln 2 - \frac{1}{4} + \frac{1}{2} - \frac{1}{2}\ln 2 = \boxed{\frac{1}{4}}$$

Opción B:

4. $f(x) = \dfrac{x+1}{e^x}$

Recta tangente en pto. de inflexión.

$y - f(x_0) = f'(x_0)[x - x_0]$ $\qquad f''(x_0) = 0$

$$f'(x) = \frac{e^x - (x+1)e^x}{e^{2x}} = -\frac{x}{e^x}$$

$$f''(x) = -\frac{e^x - xe^x}{e^{2x}} = \frac{x-1}{e^x}$$

$$f''(x) = 0 \Rightarrow \frac{x-1}{e^x} = 0 \Rightarrow x_0 = 1.$$

$f'(1) = -\frac{1}{e}$: pendiente de la recta tangente.

$f(1) = \frac{2}{e}$

$\longrightarrow$ Recta tangente: $y - \frac{2}{e} = -\frac{1}{e}(x-1)$

$$\boxed{y = \frac{-x+3}{e}}$$

5. $f(x) = x^3 - 4x$

$g(x) = 3x - 6$

a) Puntos de corte de f y g: $f(x) = g(x)$

$\Rightarrow x^3 - 4x = 3x - 6$

$x^3 - 7x + 6 = 0$ → Resolvemos por Ruffini.

$$\begin{array}{r|rrrr} & 1 & 0 & -7 & 6 \\ 2 & & 2 & 4 & -6 \\ \hline & 1 & 2 & -3 & \boxed{0} \end{array}$$

$x^3 - 7x + 6 = (x-2)(x^2+2x-3) = 0 \begin{cases} \underline{x=2} \\ x^2+2x-3=0 \end{cases}$

$x^2 + 2x - 3 = 0$

$\Rightarrow x = \dfrac{-2 \pm \sqrt{2^2 + 4 \cdot 3}}{2} \begin{cases} \dfrac{-2+4}{2} = \underline{1} \\ \dfrac{-2-4}{2} = \underline{-3} \end{cases}$

$x=2 \to f(2) = 0$

$x=1 \to f(1) = -3$

$x=-3 \to f(-3) = -15$

$\to \boxed{\left.\begin{matrix} (2,0) \\ (1,-3) \\ (-3,-15) \end{matrix}\right\} \text{Puntos de corte de } f(x) \text{ y } g(x)}$

b) $\text{Área} = \left| \int_{-3}^{1} (f-g)\,dx \right| + \left| \int_{1}^{2} (f-g)\,dx \right| = \left[\dfrac{x^4}{4} - \dfrac{7x^2}{2} + 6x \right]_{-3}^{1} - \left[\dfrac{x^4}{4} - \dfrac{7x^2}{2} + 6x \right]_{1}^{2} =$

$= \boxed{\dfrac{131}{4} \, u^2}$

$\int (x^3 - 7x + 6)\,dx = \dfrac{x^4}{4} - \dfrac{7x^2}{2} + 6x$