1. $f: (1, +\infty) \to \mathbb{R}$ $\quad f''(x) = \frac{1}{(x-1)^2}$. $\quad f'(2)$: recta tg: $y = x+2$

$\Rightarrow f'(2) = 1$

$$f(x) = \int f'(x)\,dx$$

$$f'(x) = \int f''(x)\,dx = \int \frac{1}{(x-1)^2}\,dx = \frac{-1}{x-1} + k$$

$$f'(2) = \frac{-1}{2-1} + k = 1 \Rightarrow -1 + k = 1 \Rightarrow k = 2$$

$$\Rightarrow f'(x) = \frac{-1}{x-1} + 2$$

$$f(x) = \int \left(2 - \frac{1}{x-1}\right) dx = 2x - \ln|x-1| + k'$$

Recta tg en $x=2$: $y = x+2$

$$\left.\begin{array}{l} y - y_0 = \underset{1}{\underbrace{f'(2)}}(x-2) \end{array}\right\} \Rightarrow y - y_0 = x-2$$

$$y = x - 2 + y_0 = x+2$$

$$\Rightarrow y_0 = 4 = f(2)$$

$$f(2) = 2 \cdot 2 - \ln|2-1| + k' = 4$$

$$\Rightarrow k' = 0$$

$$\boxed{f(x) = 2x - \ln|x-1|}$$

2. a) $\int x\cos\left(\frac{x}{2}\right)dx = 2x\,\text{sen}\left(\frac{x}{2}\right) - 2\int \text{sen}\left(\frac{x}{2}\right)dx = \boxed{2x\,\text{sen}\left(\frac{x}{2}\right) + 4\cos\left(\frac{x}{2}\right) + k}$

$u = x \to du = dx$

$dv = \cos\left(\frac{x}{2}\right)dx \to v = 2\,\text{sen}\left(\frac{x}{2}\right)$

b) $F(0) = 1$

$F(x) = 2x\,\text{sen}\left(\frac{x}{2}\right) + 4\cos\left(\frac{x}{2}\right) + k$

$F(0) = 4 + k \Rightarrow k = -3$

$\boxed{F(x) = 2x\,\text{sen}\left(\frac{x}{2}\right) + 4\cos\left(\frac{x}{2}\right) - 3}$

3. $f''(x) = xe^x$

$f'(1) = 0 \to$ Extremo relativo

$f(0) = 0$

$f'(x) = \int f''(x)dx = \int xe^x dx = xe^x - \int e^x dx = xe^x - e^x + k$

$u = x \to du = dx$

$dv = e^x dx \to v = e^x$

$f'(1) = e^1 - e^1 + k = 0 \Rightarrow k = 0$

$f(x) = \int f'(x)dx = \int (xe^x - e^x)dx = \int xe^x dx - \int e^x dx =$

$= xe^x - e^x - e^x + k' = xe^x - 2e^x + k'$

$f(0) = -2 + k' = 0 \Rightarrow k' = 2$

$\Rightarrow \boxed{f(x) = xe^x - 2e^x + 2}$

4. $f(x) = (x+2)\ln x$

a) $\int (x+2)\ln x \, dx = \frac{(x+2)^2}{2}\ln x - \frac{1}{2}\int \frac{(x+2)^2}{x} dx =$

$u = \ln x \rightarrow du = \frac{1}{x}dx$

$dv = (x+2)dx \rightarrow v = \frac{(x+2)^2}{2}$

$= \frac{(x+2)^2}{2}\ln x - \frac{1}{2}\int \left(x + 4 + \frac{4}{x}\right)dx = \frac{(x+2)^2}{2}\ln x - \frac{1}{4}x^2 - 2x - 2\ln x + k$

$= \boxed{\frac{x^2}{2}\ln x + 2x\ln x - \frac{1}{4}x^2 - 2x + k}$

b) $F(1) = 0$

$F(1) = \frac{1}{4} \quad -2 + k = 0 \Rightarrow k = -\frac{7}{4}$

$\boxed{F(x) = \frac{x^2}{2}\ln x + 2x\ln x - \frac{1}{4}x^2 - 2x - \frac{7}{4}}$

5. a) $\int \frac{x^2}{(1+x^3)^{3/2}} dx = \frac{1}{3}\int \frac{dt}{t^{3/2}} = \frac{1}{3}\cdot(-2)t^{-1/2} + k = \boxed{\frac{-2}{3}(1+x^3)^{-1/2} + k}$

$t = 1 + x^3 \rightarrow dt = 3x^2 dx$

b) $F(2) = 0$

$F(2) = \frac{-2}{3}\cdot(1+2^3)^{-1/2} + k = 0 \Rightarrow k = \frac{2}{9}$

$\boxed{F(x) = \frac{-2}{3}(1+x^3)^{-1/2} + \frac{2}{9}}$

6. $f(x) = x\arctan(x)$

$F(x) = \int f(x)\,dx \quad ; \quad F(0) = \pi$

$F(x) = \int x\arctan(x)\,dx = \frac{x^2}{2}\arctan(x) - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx =$

$\left\{\begin{array}{l} u = \arctan(x) \to du = \frac{1}{1+x^2}\,dx \\ dv = x\,dx \to v = \frac{x^2}{2} \end{array}\right\}$

$= \frac{x^2}{2}\arctan(x) - \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right)dx = \frac{x^2}{2}\arctan(x) - \frac{1}{2}x + \frac{1}{2}\arctan(x) + k$

$F(0) = \frac{1}{2}\arctan(0) + k = \quad + k = \pi \Rightarrow k = \pi$

$$\boxed{F(x) = \frac{x^2}{2}\arctan(x) - \frac{1}{2}x + \frac{1}{2}\arctan(x) + \pi}$$

7. $\int \frac{x}{1+\sqrt{x}}\,dx = 2\int \frac{t^3}{1+t}\,dt = 2\int\left(t^2 - t + 1 - \frac{1}{1+t}\right)dt =$

$t = \sqrt{x} \to dt = \frac{1}{2\sqrt{x}}\,dx$

$= 2\left(\frac{t^3}{3} - \frac{t^2}{2} + t - \ln|1+t|\right) + k = \boxed{\frac{2}{3}\sqrt{x^3} - x + \frac{\sqrt{x}}{2} - 2\ln|1+\sqrt{x}| + k}$

8. $\int \frac{\sqrt{2x+1}}{2x+1+\sqrt{2x+1}}\,dx = \boxed{\sqrt{2x+1} - \ln\left|\sqrt{2x+1}+1\right| + k}$

$t = \sqrt{2x+1} \to dt = \frac{1}{\sqrt{2x+1}}\,dx$

9. $f''(x) = -2\,\text{sen}(2x)$

$f(0) = 1$

$f\left(\frac{\pi}{2}\right) = 0$

$$f'(x) = \int f''(x)\,dx = -2\int \text{sen}(2x)\,dx = \cos(2x) + k$$

$$f(x) = \int f'(x)\,dx = \int (\cos(2x) + k)\,dx = \frac{1}{2}\,\text{sen}(2x) + kx + k'$$

$f(0) = \underline{k' = 1}$

$f\left(\frac{\pi}{2}\right) = k \cdot \frac{\pi}{2} + 1 = 0 \Rightarrow k = -\frac{2}{\pi}$

$$\boxed{f(x) = \frac{1}{2}\,\text{sen}(2x) - \frac{2}{\pi}x + 1}$$

10. $f(x) = \dfrac{x^2+1}{x^2(x-1)}$; $x \neq 0,\ x \neq 1$ $\qquad F(2) = \ln(2)$

$$F(x) = \int f(x)\,dx$$

a) $F'(x) = f(x)$; $F'(2) = f(2) = \dfrac{5}{4}$

Recta tangente: $\boxed{y - \ln(2) = \frac{5}{4}(x-2)}$

b) $$F(x) = \int \frac{x^2+1}{x^2(x-1)}\,dx = \int \frac{-1}{x}\,dx - \int \frac{1}{x^2}\,dx + \int \frac{2}{x-1}\,dx =$$

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} = \frac{x^2+1}{x^2(x-1)} \Rightarrow \begin{array}{l} A = -1 \\ B = -1 \\ C = 2 \end{array}$$

$F(x) = -\ln|x| + \frac{1}{x} + 2\ln|x-1| + k$; $F(2) = -\ln(2) + \frac{1}{2} + k = \ln(2)$

$$\boxed{F(x) = -\ln|x| + \frac{1}{x} + 2\ln|x-1| + 2\ln(2) - \frac{1}{2}}$$

11. $f(x) = \dfrac{\ln(x)}{2x}$; $x>0$; $F(1)=2$

$$F(x) = \int f(x)\,dx$$

a) $F'(x) = f(x)$

$$F'(e) = f(e) = \frac{1}{2e}$$

b) $F(x) = \displaystyle\int \frac{\ln(x)}{2x}\,dx = \frac{1}{2}(\ln(x))^2 - \int \frac{\ln(x)}{2x}\,dx$

$u = \ln(x) \rightarrow du = \frac{1}{x}dx$

$dv = \dfrac{dx}{2x} \rightarrow v = \frac{1}{2}\ln(x)$

$$\Rightarrow I = \frac{1}{2}(\ln(x))^2 - I \Rightarrow 2I = \frac{1}{2}(\ln(x))^2$$

$$I = \int \frac{\ln(x)}{2x}\,dx = \frac{1}{4}(\ln(x))^2 + k = F(x)$$

$F(1) = k = 0 \rightarrow F(x) = \frac{1}{4}(\ln(x))^2 \rightarrow F(e) = \frac{1}{4}$

Recta tangente: $y - F(e) = F'(e)(x-e)$

$$\boxed{y - \frac{1}{4} = \frac{1}{2e}(x-e)}$$

12. $\int e^{2x} \operatorname{sen}(x)\,dx = -e^{2x}\cos(x) + 2\int e^{2x}\cos(x)\,dx =$

$u = e^{2x} \to du = 2e^{2x}dx$ | $u = e^{2x} \to du = 2e^{2x}dx$

$dv = \operatorname{sen}(x)dx \to v = -\cos(x)$ | $dv = \cos(x)dx \to v = \operatorname{sen}(x)$

$= -e^{2x}\cos(x) + 2\left[e^{2x}\operatorname{sen}(x) - 2\int e^{2x}\operatorname{sen}(x)\,dx\right] = -e^{2x}\cos(x) + 2e^{2x}\operatorname{sen}(x) - 4\int e^{2x}\operatorname{sen}(x)dx$

$I = -e^{2x}\cos(x) + 2e^{2x}\operatorname{sen}(x) - 4I$

$\Rightarrow \boxed{I = \frac{1}{5} e^{2x}\left(-\cos(x) + 2\operatorname{sen}(x)\right) + k}$

13. $\int \frac{dx}{(x-2)\sqrt{x+2}} = 2\int \frac{dt}{t^2-4} = \frac{1}{2}\int \frac{dt}{t-2} - \frac{1}{2}\int \frac{dt}{t+2} = \frac{1}{2}\ln|t-2| - \frac{1}{2}\ln|t+2|$

$t = \sqrt{x+2} \to dt = \frac{1}{2\sqrt{x+2}}dx$ $\qquad \frac{1}{t^2-4} = \frac{A}{t-2} + \frac{B}{t+2}$ $\begin{cases} A = 1/4 \\ B = -1/4 \end{cases}$

$\downarrow$

$t^2 - 2 = x$

$\Rightarrow \boxed{\int \frac{dx}{(x-2)\sqrt{x+2}} = \frac{1}{2}\ln\left(\sqrt{x+2} - 2\right) - \frac{1}{2}\ln\left(\sqrt{x+2} + 2\right) + k}$

14. $\int \frac{-x^2}{x^2+x-2}dx = -\int\left(1-\frac{x-2}{x^2+x-2}\right)dx = -x+\int\frac{x-2}{x^2+x-2}dx =$

$x^2+x-2=(x+2)(x-1) \to \frac{A}{x+2}+\frac{B}{x-1} = \frac{x-2}{x^2+x-2}$ $\begin{cases} A = 4/3 \\ B = -1/3 \end{cases}$

$$\boxed{\int\frac{-x^2}{x^2+x-2}dx = -x-\frac{1}{3}\ln|x-1|+\frac{4}{3}\ln|x+2|+k}$$

15. $f''(x) = \ln(x)$

$f'(1) = 0$

$f(1) = 2$

$f'(x) = \int \ln(x)\,dx = x\ln|x|-x+k$

$f'(1) = -1+k = 0 \Rightarrow k = 1 \Rightarrow f'(x) = x\ln|x|-x+1$

$f(x) = \int\left(x\ln|x|-x+1\right)dx = \frac{1}{2}x^2\ln|x|-\frac{3}{4}x^2+x+k'$

$f(1) = \frac{-3}{4}+1+k' = 2 \to k' = \frac{7}{4}$

$$\Rightarrow \boxed{f(x) = \frac{1}{2}x^2\ln|x|-\frac{3}{4}x^2+x+\frac{7}{4}}$$