1. $\displaystyle\int_{-2}^{-1} \frac{dx}{(x^2-x)(x-1)}$

$$\frac{1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$A(x-1)^2 + Bx(x-1) + Cx = Ax^2 - 2Ax + A + Bx^2 - Bx + Cx = 1$$

$$(A+B)x^2 - (2A+B-C)x + A = 1$$

$$\left.\begin{array}{l} A+B=0 \rightarrow B=-1 \\ 2A+B-C=0 \\ A=1 \end{array}\right\} \rightarrow 2-1-C=0 \Rightarrow C=1$$

$$\int_{-2}^{-1} \frac{dx}{(x^2-x)(x-1)} = \int_{-2}^{-1} \frac{dx}{x} - \int_{-2}^{-1} \frac{dx}{x-1} + \int_{-2}^{-1} \frac{dx}{(x-1)^2} = \Big(\ln|x| - \ln|x-1| - (x-1)^{-1}\Big]_{-2}^{-1} =$$

$$= -\ln 2 - \ln 2 + \ln 3 + \frac{1}{2} - \frac{1}{3} = \boxed{\ln 3 + \frac{1}{6}\ u^2}$$

$$\boxed{-2\ln 2}$$

2. $f(x) = e^{-2x}$

a) Recta tangente en $x=-1/2$: $y = -2ex$

$f(-\frac{1}{2}) = e$ $\qquad y_{x=-1/2} = e$

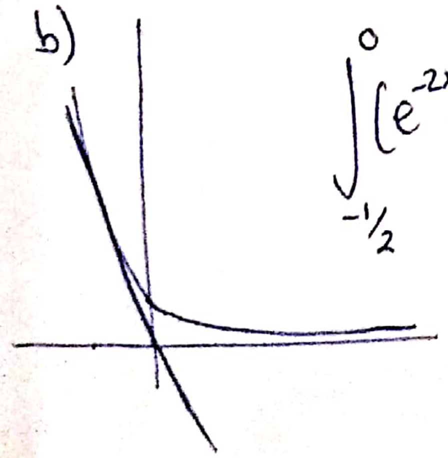
La función y la recta coinciden en el punto $(-\frac{1}{2}, e)$

Pendiente de la recta tg: $-2e$

$f'(x) = -2e^{-2x}$

$f'(-\frac{1}{2}) = -2e \Rightarrow$ La recta tangente es la dada. □

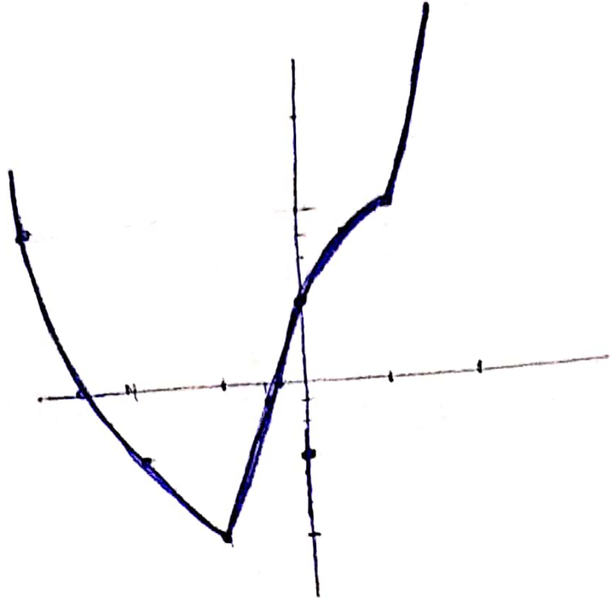
b)



$$\int_{-1/2}^{0} (e^{-2x} + 2ex)\,dx = \Big[-\frac{1}{2}e^{-2x} + ex^2\Big|_{-1/2}^{0} = -\frac{1}{2} + \frac{1}{2}e - \frac{1}{4}e = \boxed{-\frac{1}{2} + \frac{1}{4}e\ u^2}$$

3. $g(x) = 2x + |x^2 - 1|$

a) $x^2 - 1 = 0 \longrightarrow x = \pm 1$



$$g(x) = \begin{cases} 2x + x^2 - 1 \ ; & x \leq -1 \\ 2x - x^2 + 1 \ ; & -1 < x < 1 \\ 2x + x^2 - 1 \ ; & x \geq 1 \end{cases}$$

$2 \cdot \frac{1}{2} - \frac{1}{4} + 1 = 2 - \frac{1}{4} = \frac{7}{4}$

$-2 \cdot \frac{1}{2} - \frac{1}{4} + 1 = -\frac{1}{4}$

Corte con eje x: $2x - x^2 + 1 = 0$

$-x^2 + 2x + 1 = 0$

$x = \frac{-2 \pm \sqrt{4+4}}{-2} = \frac{+2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$

Corte eje x: $x^2 + 2x - 1 = 0$

$x = \frac{-2 \pm \sqrt{4+4}}{2} = -1 \pm \sqrt{2}$

$-6 + 9 - 1$

b) $\int_0^2 g(x)\,dx = \int_0^1 g(x)\,dx + \int_1^2 g(x)\,dx = \int_0^1 (2x - x^2 + 1)\,dx + \int_1^2 (2x + x^2 - 1)\,dx =$

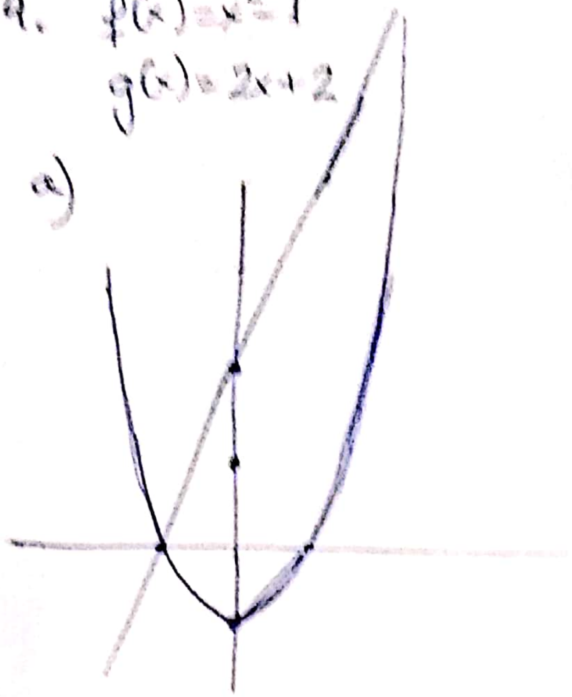
$= \left(x^2 - \frac{x^3}{3} + x \right]_0^1 + \left(x^2 + \frac{x^3}{3} - x \right]_1^2 = \cancel{1} - \frac{1}{3} + \cancel{1} + 4 + \frac{8}{3} - 2 - \cancel{1} - \frac{1}{3} + \cancel{1} =$

$= 4 + \frac{8}{3} - \frac{2}{3} = 4 + 2 = \boxed{6\,u^2}$

4. $f(x) = x^2 - 1$

$g(x) = 2x + 2$

a)



b) $f(x) = g(x)$

$x^2 - 1 = 2x + 2$

$x^2 - 2x - 3 = 0$

$x = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} \begin{cases} 3 \\ -1 \end{cases}$

$$\int_{-1}^{3} (g(x) - f(x))dx = \int_{-1}^{3} (2x + 2 - x^2 + 1)dx = \int_{-1}^{3} (2x - x^2 + 3)dx =$$

$$= \left(x^2 - \frac{x^3}{3} + 3x\right]_{-1}^{3} = \left(9 - \frac{27}{3} + 9 - 1 + \frac{-1}{3} - 3\cdot(-1)\right] =$$

$$= 9 - 1 - \frac{1}{3} + 3 = \boxed{\frac{32}{3} u^2}$$