

1. Resuelve los siguientes sistemas de ecuaciones:

$$\text{a) } \begin{cases} -x+2y-z=2 \\ x-y+2z=1 \\ -2x+z=-1 \end{cases}$$

$$\text{c) } \begin{cases} -2x+y-z=4 \\ -x-3y+z=-8 \\ -2y=-4 \end{cases}$$

$$\text{b) } \begin{cases} 3x+5y=33+2z \\ 3x=19-y \\ 10+3z=x+2y \end{cases}$$

$$\text{d) } \begin{cases} 3a+2b+2c=1 \\ 2a+b+c=0 \\ 3b+2c=-1 \end{cases}$$

$$\text{a) } x=1$$

$$y=2$$

$$z=1$$

$$\text{c) } x=0$$

$$y=2$$

$$z=-2$$

$$\text{b) } x=5$$

$$y=4$$

$$z=1$$

$$\text{d) } a=-1$$

$$b=-5$$

$$c=7$$

2. Discute los siguientes sistemas de ecuaciones lineales:

$$\text{a) } \begin{cases} x+3y-5z=-8 \\ 4x+9y-10z=-8 \\ 3x+6y-5z=0 \end{cases}$$

$$\text{c) } \begin{cases} 3a+2b-6c+3d=7 \\ a-b+2c-d=6 \\ 6a-b=3 \end{cases}$$

$$\text{b) } \begin{cases} 8x-6y+2z=-1 \\ 3x+y-z=10 \\ x-3y+2z=-5 \end{cases}$$

$$\text{d) } \begin{cases} 3a+2b-6c+3d=7 \\ a-b+2c-d=6 \\ 7a+b-6c+3d=32 \end{cases}$$

$$\text{a) } \text{Rango}(A) = 2 = \text{Rango}(A') \neq 3 = n \Rightarrow \text{SCI}$$

$$\text{b) } \text{Rango}(A) = 3 = \text{Rango}(A') = 3 \Rightarrow \text{SCD}$$

$$\text{c) } \text{Rango}(A) = 2 = \text{Rango}(A') = 3 \Rightarrow \text{SI}$$

$$\text{d) } \text{Rango}(A) = 3 = \text{Rango}(A') \neq 4 = n \Rightarrow \text{SCI}$$

3. Discute los siguientes sistemas de ecuaciones en función del parámetro m :

a)
$$\begin{cases} x+2y+z=m \\ 2x+3y+z=m \\ x+y-mz=m \end{cases}$$

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & -m \end{pmatrix}$$

$$A' = \begin{pmatrix} 1 & 2 & 1 & m \\ 2 & 3 & 1 & m \\ 1 & 1 & -m & m \end{pmatrix}$$

$$|A| = m$$

$$\text{Si } m \neq 0 \Rightarrow \text{Rango}(A) = \text{Rango}(A') = 3 = n \Rightarrow \text{S.C.D.}$$

$$\text{Si } m = 0 \Rightarrow \text{Rango}(A) = \text{Rango}(A') = 2 < n \Rightarrow \text{S.C.I.}$$

$$\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} \neq 0$$

b)
$$\begin{cases} 2x+y+3z=2 \\ 5x+2y+4z=-1 \\ 3x+y+m^2z=3m \end{cases}$$

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 5 & 2 & 4 \\ 3 & 1 & m^2 \end{pmatrix}$$

$$A' = \begin{pmatrix} 2 & 1 & 3 & 2 \\ 5 & 2 & 4 & -1 \\ 3 & 1 & m^2 & 3m \end{pmatrix}$$

$$|A| = 1 - m^2$$

$$\text{Si } m \in \mathbb{R} - \{-1, 1\} \Rightarrow \text{Rango}(A) = \text{Rango}(A') = 3 = n \Rightarrow \text{S.C.D.}$$

$$\text{Si } m = -1 \Rightarrow \text{Rango}(A) = 2 = \text{Rango}(A') = 2 < n \Rightarrow \text{S.C.I.}$$

$$\text{Si } m = 1 \Rightarrow \text{Rango}(A) = 2 \neq \text{Rango}(A') = 3 \Rightarrow \text{S.I.}$$

c)
$$\begin{cases} -x-y+3z=0 \\ 2x+mz=0 \\ 2y+4z=0 \end{cases}$$

$$A = \begin{pmatrix} -1 & -1 & 3 \\ 2 & 0 & m \\ 0 & 2 & 4 \end{pmatrix}$$

$$A' = \begin{pmatrix} -1 & -1 & 3 & 0 \\ 2 & 0 & m & 0 \\ 0 & 2 & 4 & 0 \end{pmatrix}$$

$$|A| = 2m + 20$$

$$\text{Si } m \neq -10 \Rightarrow \text{Rango}(A) = \text{Rango}(A') = 3 = n \Rightarrow \text{S.C.D.}$$

$$\text{Si } m = -10 \Rightarrow \text{Rango}(A) = \text{Rango}(A') = 2 \Rightarrow \text{S.C.I.}$$

↑
Sist. Homógeno
→ SIEMPRE es
COMPATIBLE

d)
$$\begin{cases} x+y+z=5 \\ 5x+y-z=11 \\ 3x-y+mz=2 \end{cases}$$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 5 & 1 & -1 \\ 3 & -1 & m \end{pmatrix}$$

$$A' = \begin{pmatrix} 1 & 1 & 1 & 5 \\ 5 & 1 & -1 & 11 \\ 3 & -1 & m & 2 \end{pmatrix}$$

$$|A| = -4m - 12$$

$$\text{Si } m \neq -3 \Rightarrow \text{Rango}(A) = \text{Rango}(A') = 3 = n \Rightarrow \text{S.C.D.}$$

$$\text{Si } m = -3 \Rightarrow \text{Rango}(A) = 2 \neq \text{Rango}(A') \Rightarrow \text{S.I.}$$

4. Resuelve en función del parámetro m:

$$\begin{cases} -x+y-z=-1 \\ 4x-2y+2z=2m \\ -3x-2y+mz=-4 \end{cases}$$

$$A = \begin{vmatrix} -1 & 1 & -1 \\ 4 & -2 & 2 \\ -3 & -2 & m \end{vmatrix} = 4-2m$$

$$\begin{cases} \rightarrow \text{Si } m=2 \Rightarrow |A|=0 \\ \rightarrow \text{Si } m \neq 2 \Rightarrow |A| \neq 0 \end{cases}$$

$$x = \frac{\begin{vmatrix} -1 & 1 & -1 \\ 2m & -2 & 2 \\ -4 & -2 & m \end{vmatrix}}{|A|} = \frac{-2m^2+6m-4}{4-2m} = \frac{-2(1-m)(2-m)}{2(2-m)} = -(1-m) = \underline{\underline{m-1}}$$

$$y = \frac{\begin{vmatrix} -1 & -1 & -1 \\ 4 & 2m & 2 \\ -3 & -4 & m \end{vmatrix}}{|A|} = \frac{-2(m^2+m-7)}{2(2-m)} = -\frac{m^2+m-7}{2-m} = \underline{\underline{\frac{m^2+m-7}{m-2}}}$$

$$z = \frac{\begin{vmatrix} -1 & 1 & -1 \\ 4 & -2 & 2m \\ -3 & -2 & -4 \end{vmatrix}}{|A|} = \frac{22-10m}{2(2-m)} = \underline{\underline{\frac{11-5m}{2-m}}}$$