1. $A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

a) $A^2 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \underline{\underline{2I}}$.

A^{-1}: $|A| = -2$

$Adj(A) = \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$

$(Adj(A))^t = \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$

$\Bigg\} \underline{\underline{A^{-1}}} = \frac{1}{|A|}(Adj(A))^t = \underline{\underline{\begin{pmatrix} +1/2 & +1/2 \\ +1/2 & -1/2 \end{pmatrix}}} = \frac{1}{2}A$

b) A^{2013}:

$A^2 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2I = 2^{\frac{2}{2}}I$

$A^3 = A^2 \cdot A = 2IA = 2A$

$A^4 = A^3 \cdot A = 2A \cdot A = 2A^2 = 2 \cdot 2I = 4I = 2^2 I = 2^{\frac{4}{2}} I$

$A^5 = 4A$

$A^6 = 8I = 2^3 I = 2^{\frac{6}{2}} I$

$\frac{2013}{2} = 1006 + \frac{1}{2}$

$\underline{\underline{A^{2013}}} = A^{2012} \cdot A = 2^{\frac{2012}{2}} \cdot I \cdot A = \underline{\underline{2^{1006} A}}$

$(A^{2013})^{-1}$:

$|A^{2013}| = (2^{1006})^2 \cdot |A|$

$\underline{\underline{(A^{2013})^{-1}}} = \frac{1}{2^{2012}}(A^{-1}) = \underline{\underline{\frac{1}{2^{2012}}\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{pmatrix}}} = \underline{\underline{\frac{1}{2^{2013}} \cdot A}}$

2. $A = \begin{pmatrix} -1 & 1 & 0 \\ 2 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$; $B = \begin{pmatrix} 0 & 2 & 1 \\ 1 & 2 & 0 \end{pmatrix}$; $C = \begin{pmatrix} 1 & 2 \\ -1 & 6 \end{pmatrix}$

a) A^{-1}:

$|A| = -2$

$Adj(A) = \begin{pmatrix} 0 & -2 & 0 \\ -1 & -1 & +1 \\ 0 & 0 & -2 \end{pmatrix}$

$(Adj(A))^t = \begin{pmatrix} 0 & -1 & 0 \\ -2 & -1 & 0 \\ 0 & 1 & -2 \end{pmatrix}$

$\Bigg\} A^{-1} = \frac{1}{|A|}(Adj(A))^t = \underline{\underline{\begin{pmatrix} 0 & +1/2 & 0 \\ 1 & 1/2 & 0 \\ 0 & -1/2 & 1 \end{pmatrix}}}$

b) $AX = B^tC \Rightarrow X = A^{-1}B^tC$

$$B^tC = \begin{pmatrix} 0 & 1 \\ 2 & 2 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & 6 \end{pmatrix} = \begin{pmatrix} -1 & 6 \\ 0 & 16 \\ 1 & 2 \end{pmatrix}$$

$$A^{-1}B^tC = \begin{pmatrix} 0 & 1/2 & 0 \\ 1 & 1/2 & 0 \\ 0 & -1/2 & 1 \end{pmatrix}\begin{pmatrix} -1 & 6 \\ 0 & 16 \\ 1 & 2 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 0 & 8 \\ -1 & 14 \\ 1 & -6 \end{pmatrix} = X}}$$

c) $$|A^{2013} \cdot B^t \cdot B \cdot (A^{-1})^{2013}| = |A^{2013}| \cdot |B^t \cdot B| \cdot |(A^{-1})^{2013}| = |A^{2013}| \cdot |B^tB| \cdot \frac{1}{|A^{2013}|} = |B^t \cdot B|$$

$$B^t \cdot B = \begin{pmatrix} 0 & 1 \\ 2 & 2 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 2 & 1 \\ 1 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 8 & 2 \\ 0 & 2 & 1 \end{pmatrix} \Rightarrow |B^tB| = 0$$

$$\Rightarrow |A^{2013} B^t B (A^{-1})^{2013}| = \underline{\underline{0}}$$

3. $|A| = 4$

a) $\det(-2A) = (-2)^3 \cdot |A| = -8 \cdot 4 = \underline{\underline{-32}}$

(A es de orden 3)

$\underline{\underline{\det(A^{-1})}} = \frac{1}{|A|} = \underline{\underline{\frac{1}{4}}}$

b) $$\begin{vmatrix} a & -b & c \\ 2d & -2e & 2f \\ p & -q & r \end{vmatrix} = 2\begin{vmatrix} a & -b & c \\ d & -e & f \\ p & -q & r \end{vmatrix} = 2 \cdot (-1) \cdot \begin{vmatrix} a & b & c \\ d & e & f \\ p & q & r \end{vmatrix} = 2(-1)|A| = \underline{\underline{-8}}$$

$$\begin{vmatrix} -3d & -3e & -3f \\ a & b & c \\ -p & -q & -r \end{vmatrix} = -3 \cdot \begin{vmatrix} d & e & f \\ a & b & c \\ -p & -q & -r \end{vmatrix} = -3 \cdot (-1)\begin{vmatrix} d & e & f \\ a & b & c \\ p & q & r \end{vmatrix} = -3 \cdot (-1) \cdot (-1)\begin{vmatrix} a & b & c \\ d & e & f \\ p & q & r \end{vmatrix} = -3 \cdot 4 = \underline{\underline{-12}}$$

(intercambio de filas)

4. $A = \begin{pmatrix} -1 & 2 \\ 2 & m \end{pmatrix}$ $\quad B = \begin{vmatrix} -1 & 2 & 0 \\ -2 & m & 0 \\ 3 & 2 & m \end{vmatrix}$

a) Para que tengan el mismo rango, rango $(B) \neq 3 \Rightarrow |B| = 0$

$$|B| = -m^2 + 4m = 0 \Rightarrow \begin{cases} m = 0 \\ m = 4 \end{cases}$$

$m = 0$: rango $(A) = 2$ $\quad \begin{vmatrix} -1 & 2 \\ 2 & 0 \end{vmatrix} = -4 \neq 0$

rango $(B) = 2$ $\quad \begin{vmatrix} -1 & 2 \\ -2 & 0 \end{vmatrix} = 4 \neq 0$

$m = 4$: rango $(A) = 2$ $\quad \begin{vmatrix} -1 & 2 \\ 2 & 4 \end{vmatrix} = -8 \neq 0$

rango $(B) = 2$ $\quad \begin{vmatrix} -2 & 4 \\ 3 & 2 \end{vmatrix} = -16 \neq 0$

$\Rightarrow$ Si $\underline{m = 0}$ ó $\underline{m = 4}$, rango $(A) =$ rango (B)

b) $|A| = -m - 4$

$|B| = -m^2 + 4m$

$|A| = |B| \Rightarrow -m - 4 = -m^2 + 4m$

$$m^2 - 5m - 4 = 0$$

$$m = \frac{5 \pm \sqrt{25 + 16}}{2} = \frac{5 \pm \sqrt{41}}{2}$$

$\underline{m = \frac{5 + \sqrt{41}}{2}}$

$\underline{m = \frac{5 - \sqrt{41}}{2}}$

Valores de m para los que los determinantes son iguales.